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Supplemental Remarks on Spin-Dependent Interactions in Elastic Scattering of Deuterons at Intermediate Energies

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Abstract

For elastic scattering of deuterons at intermediate energies, spin-space tensor amplitudes are introduced by the invariant-amplitude method so that effects of T_R-and T_L-type tensor interactions are described separately. Quantum-mechanical corrections to the classical concept of angular momenta are shown to vanish in the derivation of the spin-space tensor amplitudes. The expressions of cross section for unpolarized beam and analyzing powers and polarization transfer coefficients for polarized beam are given in terms of the spin-space tensor amplitudes.

§ 1. Introduction

Interactions of deuterons with nuclei are characterized by having tensor components in addition to central and spin-orbit ones. The latters are familiar in nucleon-nucleus interactions. For the tensor interaction, three types were proposed earlier in the viewpoint of phenomenology[1], the coordinate-dependent one T_R, the angular-momentum-dependent one T_L and the momentum-dependent one T_P. In actual scattering or reactions, however, effects of these interactions are usually mixed up with each other even in one observable and it is hard to distinguish the effect of a particular tensor interaction from those of the others. In the following, the consideration will be focussed on the effects of the tensor forces in the elastic scattering by spherical nuclei.

Recently, by the use of the adiabatic approximation and the two-step model, the T_R-, T_L-and T_P-type tensor interactions have been derived[2,3] as the second-order effect of the nucleon-nucleus spin-orbit interactions. The result is applicable to intermediate-energy scattering because of the nature of the approximation used. There, contributions of the T_P-interaction to polarization observables have been found to be small. On the other hand, qualitative estimations have indicated that other two tensor interactions make significant contributions to the observables. Thus, the second-order effect of the spin-orbit interaction possibly provides an important correction to the T_R-type tensor interaction which has conventionally been attributed to the effect of the D-state admixture in the deuteron ground-state wave function[4], and also it gives the sound basis to the phenomenological proposal of the T_L-type tensor interaction, the origin of which has been unclear. To investigate these contributions quantitatively, it will be worthwhile to find a method which identifies the effect of the T_R-tensor interaction and that of the T_L-tensor one, separately.

The method in the case of the elastic scattering has been proposed[5,2] by the present authors and their collaborators but it has essentially assumed the validity of the classical concept of the orbital angular momentum and the quantum-mechanical effect has little been investigated. One of the purposes of the present article is to describe the details of the quantum-mechanical effect and to give the justification of the validity of the classical concept. The theory is based on the invariant-amplitude method, which was proposed earlier[6] as a tool of the examination of roles of spin-dependent interactions in polarization phenomena. There, the scattering amplitude is decomposed into the invariant amplitudes classified according to their tensorial characters in the spin space and thus it is expected to be easy to identify the contribution of each spin-dependent interaction separately. For the deuteron elastic scattering, we will give a short review of this method preceding to the consideration of the quantum-mechanical effect.

For the practical purpose, it will be desirable to represent the physical quantities in terms of spin-space tensor amplitudes, which are linear combinations of the invariant amplitudes convenient for the separation of the T_R-and T_L-effects, because they provide the information which observable is a good measure of a particular spin-dependent interaction. Such representations will be given for cross section for unpolarized deuteron beam and analyzing powers and polarization transfer coefficients for polarized beam. This is the second purpose of this article.

§ 2. Invariant-amplitude method in deuteron elastic scattering and quantum-mechanical contributions to angular momenta in the spin-space amplitudes

In this section, we will follow first the theoretical development on the introduction of the spin-space tensor amplitudes in ref. 2. After that the quantum-mechanical corrections will be investigated. The transition matrix of the deuteron elastic scattering from spin-less nuclei is given by designating the row and column by the z component ν of the deuteron spin as[7]

$$M=\begin{pmatrix} A & B & C \\ D & E & -D \\ C & -B & A \end{pmatrix}, \tag{2.1}$$

where the row denotes the initial state, $\nu_i=1, 0, -1$ from left to right and the column the final states, $\nu_f=1, 0, -1$ form top to bottom. The elastic scattering restricts the matrix elements as

$$C=A-E-\sqrt{2}\,(B+D)\cot\theta. \tag{2.2}$$

The matrix elements in the plane-wave states, $A\sim E$ describe the exact scattering amplitude in the non-relativistic form. To decompose the amplitudes according to the tensorial property in the spin space, we will expand M into the spin-space tensor operators, the κ component of the rank K tensor being denoted by $S_{K\kappa}$,

$$M=\sum_{K\kappa}(-)^{\kappa}S_{K-\kappa}R_{K\kappa}, \tag{2.3}$$

where $R_{K\kappa}$ is the counter part, the coordinate-space tensor. By taking the matrix element of (2.3), we get[6)]

$$\begin{aligned}&<\nu_f\,;\,\boldsymbol{k}_f|M|\nu_i\,;\,\boldsymbol{k}_i> \\ &=\sum_{K}(-)^{1-\nu_f}(11\nu_i-\nu_f|K\kappa)\sum_{r=\bar{K}-K}^{K}[C_r(\hat{\boldsymbol{k}}_i)\times C_{\bar{K}-r}(\hat{\boldsymbol{k}}_f)]_{K\kappa} \\ &\times F_{Kr}(E,\cos\theta),\end{aligned} \tag{2.4}$$

where ν's are the z components of the deuteron spin, $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are the initial and final momenta and $\hat{\boldsymbol{k}}_i$ and $\hat{\boldsymbol{k}}_f$ are their solid angles. The quantity $\bar{K}$ is K for K=even and $K+1$ for K=odd. In (2.4), the geometrical factor of the matrix element of $S_{K-\kappa}$ appears as the CG coefficient and that of $R_{K\kappa}$ is described by the κ component of the rank K tensor in the ordinary space is constructed by $C_r(\hat{\boldsymbol{k}}_i)$ and $C_{r'}(\hat{\boldsymbol{k}}_f)$. The remaining factor $F_{Kr}(E,\cos\theta)$ is invariant under the rotation of the coordinate axis and thus called as the invariant amplitude. The amplitude is designated by K, which is the rank of the associated spin-space tensor operator, and is a function of the CM energy E and the scattering angle θ but, in the following, we will skip these arguments for simplicity.

Using (2.4), one gets in the reference frame, $z /\!/ \boldsymbol{k}_i$ and $y /\!/ \boldsymbol{k}_i\times\boldsymbol{k}_f$,

$$\begin{aligned}A&=\sqrt{\frac{1}{3}}F_{00}+\sqrt{\frac{1}{24}}(3\cos^2\theta-1)F_{20}+\frac{1}{3}\cos\theta\,F_{21}+\sqrt{\frac{1}{6}}F_{22},\\ B&=\sqrt{\frac{1}{8}}\sin\theta\,F_{11}+\sqrt{\frac{3}{4}}\cos\theta\sin\theta\,F_{20}+\sqrt{\frac{1}{8}}\sin\theta\,F_{21},\\ C&=\sqrt{\frac{3}{8}}\sin^2\theta\,F_{20},\\ D&=-\sqrt{\frac{1}{8}}\sin\theta\,F_{11}+\sqrt{\frac{3}{4}}\cos\theta\sin\theta\,F_{20}+\sqrt{\frac{1}{8}}\sin\theta\,F_{21},\\ E&=\sqrt{\frac{1}{3}}F_{00}-\sqrt{\frac{1}{6}}(3\cos^2\theta-1)F_{20}-\frac{2}{3}\cos\theta\,F_{21}-\sqrt{\frac{2}{3}}F_{22}.\end{aligned} \tag{2.5}$$

Eq. (2.2) gives

$$F_{22}=F_{20}. \tag{2.6}$$

Thus we can choose four independent amplitudes, i. e. one scalar, one vector and two tensor ones. Referring to the consideration on the second-rank tensor amplitudes given later, we will define the four amplitudes as

$$U\equiv 2A+E=\sqrt{3}\,F_{00}, \tag{2.7}$$

$$S\equiv B-D=\sqrt{\frac{1}{2}}\sin\theta\,F_{11}, \tag{2.8}$$

$$T_\alpha\equiv B+D=\sqrt{\frac{1}{2}}\sin\theta(\sqrt{6}\cos\theta\,F_{20}+F_{21}), \tag{2.9}$$

$$T_\beta\equiv C+\sqrt{\frac{1}{8}}(B+D)\cot\frac{\theta}{2}=\frac{1}{2}(\sqrt{6}\,F_{20}+F_{21})\cos^2\frac{\theta}{2}, \tag{2.10}$$

where U is proportional to F_{00} and is the scalar amplitude, S is proportional to F_{11} and is the vector one and T_α and T_β consist of F_{20} and F_{21} and are the tensor ones. The scalar amplitude is associated with the scalar interaction in the spin space. Thus U describes the scattering amplitude due to the central interaction in the sense of the effective interaction. Other amplitudes, S, T_α and T_β are similarly related to the respective spin-dependent interactions. Among these amplitudes, the magnitude of U is expected to be the largest because the central interaction is usually stronger than spin-dependent interactions. As will be discussed below, T_α and T_β describe the effect of the $\mathrm{T_R}$-type tensor interaction and that of the $\mathrm{T_L}$-type one separately in the high-energy limit. Hereafter, we will call these amplitudes $U\sim T_\beta$ as the spin-space tensor amplitudes.

The $\mathrm{T_R}$-and $\mathrm{T_L}$-type tensor interactions are defined as[1]

$$\mathrm{T_R}=(S_2(\boldsymbol{s},\boldsymbol{s})\cdot R_2(\boldsymbol{R},\boldsymbol{R}))U_{\mathrm{R}}(R), \tag{2.11}$$

$$\mathrm{T_L}=(S_2(\boldsymbol{s},\boldsymbol{s})\cdot R_2(\boldsymbol{L},\boldsymbol{L}))U_{\mathrm{L}}(R), \tag{2.12}$$

where S_2 and R_2 are the second-rank tensor operators constructed by their arguments, $\boldsymbol{s}$, $\boldsymbol{R}$ and $\boldsymbol{L}$ being the deuteron spin, its space coordinate from the target and the corresponding angular momentum. To define the two tensor amplitudes so as to describe the scattering by the T_R-tensor interaction and that by the T_L-tensor one separately, we will introduce the plane-wave Born approximation (PWBA) and a classical concept for the angular momentum. Later, the quantum-mechanical corrections are investigated. However, the final justification is provided by the quantitative numerical calculations, which has been given in ref. 2. The PWBA provides the following relation[8] for the T_R-type tensor interactions,

$$\sqrt{6}\,F_{20}=-F_{21} \tag{2.13}$$

for a projectile of any spin. This relation has been numerically examined for scattering of ^{7}Li by ^{58}Ni and has been found to be valid even at $E_{lab}=20$ MeV in a good approximation[8]. Due to (2.10) and (2.13), $T_\beta=0$ for T_R tensor interactions at high energies.

Let us consider the property of the T_L-type tensor interaction in a simple way, that is, treat the operator $\boldsymbol{L}$ in the framework of the classical concept. The q component of the space tensor $R_2(\boldsymbol{L},\boldsymbol{L})$ in (2.12) is written explicitly as

$$R_{2q}=\sum_m (11\,m\,m'|2q)L_m\,L_{m'}. \tag{2.14}$$

Since $\boldsymbol{L}$ is perpendicular to the momentum in the classical concept, $L_0=0$, and we get

$$q\neq\pm1, \tag{2.15}$$

which leads to

$$\nu_f-\nu_i\neq\pm1 \tag{2.16}$$

for

$$<\nu_f|S_{2,-q}|\nu_i>\neq 0.$$

The restriction on ν in (2.16) means that for the T_L-type tensor interaction

$$B=D=0. \tag{2.17}$$

Combining this to (2.9),

$$T_\alpha=0 \tag{2.18}$$

for the $\mathrm{T_L}$-type tensor interaction.

Therefore, when only the two types, $\mathrm{T_R}$ and $\mathrm{T_L}$, are considered as tensor interactions, one will distinguish approximately the effect of one type from that of the other type by using T_α and T_β; i. e. T_α describes dominantly the effect of the $\mathrm{T_R}$-type interactions while T_β describes that of the $\mathrm{T_L}$-type ones. The quantum-mechanical treatment of $\boldsymbol{L}$ will produce a correction to (2.17); that is, the $\mathrm{T_L}$-type tensor interaction may possibly contribute to the matrix elements, B and D, due to the correction. The corrections are estimated by the PWBA. They are found to be asymptotically small by a factor $1/L$ for each partial wave of the angular momentum L. Moreover, the correction vanishes in $B+D$ because of the opposite sign of the correction term between B and D. The proof is given below.

The matrix element of the $\mathrm{T_L}$ interaction between the plane-wave states, f and i, is given by

$$f_{\nu_f\nu_i}=\langle\chi_{\nu_f}e^{i\boldsymbol{k}_f\boldsymbol{R}}|\mathrm{T_L}|\chi_{\nu_i}e^{i\boldsymbol{k}_i\boldsymbol{R}}\rangle, \tag{2.19}$$

where χ's are the spin function of the deuteron and $\boldsymbol{k}_i(\boldsymbol{k}_f)$ and $\nu_i(\nu_f)$ are the CM momentum of the deuteron and the z component of the spin in the initial (final) state. Inserting (2.12) into (2.19),

$$\begin{aligned} f_{\nu_f\nu_i}&=\sum_q(-)^q\langle\chi_{\nu_f}|S_{2-q}|\chi_{\nu_i}\rangle \\ &\quad\times\langle e^{i\boldsymbol{k}_f\boldsymbol{R}}|\sum_{\mu'}(1\,1\mu\mu'|2q)L_\mu L_{\mu'}U_\mathrm{L}(R)|e^{i\boldsymbol{k}_i\boldsymbol{R}}\rangle \\ &=\sum_q(-)^q\frac{1}{\sqrt{3}}(1||S_2||1)(12\nu_i-q|1\nu_f)\sum_{\mu'}(1\,1\mu\mu'|2q)I_{\mu\mu'}, \end{aligned} \tag{2.20}$$

where

$$I_{\mu\mu'}=\frac{1}{(2\pi)^3}\int e^{-i\boldsymbol{k}_f\boldsymbol{R}}L_\mu L_{\mu'}U_\mathrm{L}(R)e^{i\boldsymbol{k}_i\boldsymbol{R}}d\boldsymbol{R}. \tag{2.21}$$

Adopting the coordinate axis same as before and using

$$e^{i\boldsymbol{k}_i\boldsymbol{R}}=\sqrt{4\pi}\sum_{L'}(i)^{L'}\sqrt{2L'+1}\,j_{L'}(k_iR)Y_{L'0}(\hat{\boldsymbol{R}})$$

and

$$e^{-ik_fR}=4\pi\sum_{LM}(-i)^L j_L(k_fR)Y_{LM}(\hat{k}_f)Y_{LM}^*(\hat{R}),$$

we get

$$I_{\mu\mu'}=\frac{1}{(2\pi)^{3/2}}\sum_{LM}\sum_{L'}(i)^{L'-L}\sqrt{2L'+1}\,Y_{LM}(\hat{k}_f)<LM|L_\mu L_{\mu'}|L'\,0>$$
$$\times\int j_L(k_fR)U_L(R)j_{L'}(k_iR)R^2dR. \tag{2.22}$$

Further,

$$<LM|L_\mu L_{\mu'}|L'\,0>=L(L+1)(L'\,1\,0\,\mu'|L'\mu')(L'\,1\,\mu'\mu|LM)\delta_{L'L}, \tag{2.23}$$

which leads to

$$I_{\mu\mu'}=\frac{1}{(2\pi)^{3/2}}\sum_{L}\sqrt{2L+1}\,L(L+1)(L\,1\,0\,\mu'|L\mu')(L\,1\,\mu'\mu|LM)$$
$$\times Y_{L\mu+\mu'}(\hat{k}_f)\int j_L(k_fR)U_L(R)j_L(k_iR)R^2dR. \tag{2.24}$$

For $(L\,1\,0\,\mu'|L\mu')\neq 0$, the allowed μ' is

$$\mu'=\pm 1.$$

Furthermore, the classical concept of L restricts μ by $q\neq\pm 1$, i. e.

$$\mu=\pm 1.$$

Therefore, the quantum-mechanical correction arises from

$$\mu=0 \quad \text{with} \quad \mu'=\pm 1, \tag{2.25}$$

where $\mu'=-1$ is for B and $\mu'=1$ is for D. The CG coefficients appearing in (2.20) and (2.24) are totally unchanged for changing of the sign of μ' and $Y_{L\mu'}(\hat{k}_f)$ gives the opposite sign for $\mu'=\pm 1$. Then, the quantum-mechanical correction has the same value but the opposite sign for B and D. This means that the correction is zero for $B+D$ in the PWBA and qualifies that T_a is little affected by this interaction. Further, using the asymptotic form of Y_{LM} for large L, one can derive that the correction term itself is smaller by $1/L$ for each partial wave than the classically allowed terms; μ and $\mu'=\pm 1$. It should be noted that the present theoretical development can be extended so as to include spin-independent distortions in the initial and final waves, by replacing $j_L(k_i R)$ and $j_L(k_f R)$ with their respective distorted radial wave functions. This will qualify the wider validity for the conclusions obtained above.

Similar considerations can be applied to another problem. Neglecting the spin dependence of the propagator, it is shown that the second-order contribution of the T_L-type tensor interaction does not form S, the vector amplitude, in the classical limit. When the quantum-mechanical corrections are taken into account, one can prove that the contributions of the correction terms are cancelled with each other in each of B and D because the correction from $\mu'=1$ and that from $\mu'=-1$ have the same magnitude but the opposite sign. Thus, in the present approximations, the above conclusion based on the classical concept does hold even if the quantum-mechanical corrections are included.

Finally, the following should be emphasized. In ref. 2, numerical calculations confirm for some typical cases that the contribution of the T_L interaction to $B+D$ is quantitatively small.

§ 3. Expressions of analyzing powers and polarization transfer coefficients by the spin-space tensor amplitudes

Since the spin-space tensor amplitudes, U, S, T_α and T_β describe the effects of the central, spin-orbit, T_R-type tensor and T_L-type tensor interactions separately as shown in the preceding section, here we will express physical observables in terms of these amplitudes. These expressions are useful in finding the effect of the above spin-dependent interactions in the observables. In the viewpoint of the recent progress in the experimental technique on the polarized deuteron beam, we will treat the cross section by the unpolarized beam and the analyzing powers and polarization transfer coefficients by the polarized beam. They are defined below.[9]

Using the scattering T-matrix M defined in the preceding section, the cross section σ, the vector analyzing power A_y and the tensor analyzing powers A_{xx}, A_{yy} and A_{zz} are given by

$$\sigma=\frac{1}{3}N, \tag{3.1}$$

$$A_y=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_y M^+), \tag{3.2}$$

$$A_{ij}=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_{ij} M^+) \tag{3.3}$$

with

$$N=\mathrm{Tr}(MM^+), \tag{3.4}$$

where i and j describe one of x, y, z. The polarization transfer coefficients $K_i{}^j$, $K_{ij}{}^k$, $K_i{}^{jk}$ and $K_{ij}{}^{kl}$ (i, j, k and l are one of x, y, z) are given by

$$K_i{}^j=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_i M^+\mathcal{P}_j), \tag{3.5}$$

$$K_{ij}{}^k=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_{ij} M^+\mathcal{P}_k), \tag{3.6}$$

$$K_i{}^{jk}=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_i M^+\mathcal{P}_{jk}), \tag{3.7}$$

$$K_{ij}{}^{kl}=\frac{1}{N}\mathrm{Tr}(M\mathcal{P}_{ij} M^+\mathcal{P}_{kl}). \tag{3.8}$$

The spin operator $\mathcal{P}_i$ and $\mathcal{P}_{ij}$ are given in ref. 9.

Using (2.1), (2.5) and (2.7)~(2.10), the cross section, the analyzing powers and the polarization transfer coefficients are expressed by the spin-space tensor amplitudes as follows.

$$\sigma=\frac{1}{9}\left\{|U|^2+3|S|^2+8|T_\beta|^2+\frac{4}{\sin^2\theta}|T_\alpha|^2-\frac{4\sqrt{2}}{\sin\theta}Re(T_\alpha{}^*T_\beta)\right\}, \tag{3.9}$$

$$A_y=\frac{2\sqrt{2}}{3N}Im\left\{(U-2T_\beta+\frac{1}{\sqrt{2}\sin\theta}T_\alpha)S^*\right\}, \tag{3.10}$$

$$A_{xx}=\frac{1}{N}\left\{\frac{4}{3}Re(UT_\beta^*)-\sqrt{2}\,\frac{1+3\cos\theta}{3\sin\theta}Re(UT_\alpha^*)\right.$$

$$+\frac{4}{3}|T_\beta|^2-\frac{1+3\cos\theta}{3\sin^2\theta}|T_\alpha|^2-\frac{1}{2}|S|^2$$

$$\left.-2\sqrt{2}\,\frac{1-3\cos\theta}{3\sin\theta}Re(T_\alpha T_\beta^*)+3Re(ST_\alpha^*)\right\}, \tag{3.11}$$

$$A_{yy}=\frac{1}{N}\left\{-\frac{8}{3}Re(UT_\beta^*)+\frac{2\sqrt{2}}{3\sin\theta}Re(UT_\alpha^*)-\frac{8}{3}|T_\beta|^2\right.$$

$$\left.+\frac{2}{3\sin^2\theta}|T_\alpha|^2+|S|^2+\frac{4\sqrt{2}}{3\sin\theta}Re(T_\alpha T_\beta^*)\right\}, \tag{3.12}$$

$$A_{xz}=\frac{\sqrt{2}}{N}Re\left\{\left(U+\frac{3}{\sqrt{2}}S\cot\theta-2T_\beta+\frac{1}{\sqrt{2}\sin\theta}T_\alpha\right)T_\alpha^*\right\}, \tag{3.13}$$

$$K_x^x=\frac{2}{9N}\left\{|U|^2+2Re(UT_\beta^*)-\frac{1}{\sqrt{2}}\frac{1+3\cos\theta}{\sin\theta}Re(UT_\alpha^*)\right.$$

$$\left.-8|T_\beta|^2+\frac{3\cos\theta-1}{\sin^2\theta}|T_\alpha|^2+2\sqrt{2}\,\frac{2-3\cos\theta}{\sin\theta}Re(T_\alpha T_\beta^*)\right\}, \tag{3.14}$$

$$K_z^z=\frac{2}{9N}\left\{|U|^2+2Re(UT_\beta^*)+\frac{3\cos\theta-1}{\sqrt{2}\sin\theta}Re(UT_\alpha^*)-8|T_\beta|^2\right.$$

$$\left.-\frac{1+3\cos\theta}{\sin^2\theta}|T_\alpha|^2+2\sqrt{2}\,\frac{3\cos\theta+2}{\sin\theta}Re(T_\alpha T_\beta^*)\right\}, \tag{3.15}$$

$$K_x^z=\frac{\sqrt{2}}{3N}Re\{\alpha(T_\alpha+S)^*\}, \tag{3.16}$$

$$K_z^x=\frac{\sqrt{2}}{3N}Re\{\alpha(T_\alpha-S)^*\}, \tag{3.17}$$

where α is given by

$$\alpha=U+4T_\beta-\frac{\sqrt{2}}{\sin\theta}T_\alpha. \tag{3.18}$$

$$K_{xy}^z=\frac{2}{N}Im\left\{UT_\beta^*-\frac{1}{2\sqrt{2}}\frac{1+\cos\theta}{\sin\theta}UT_\alpha^*+\sqrt{2}\cot\theta\;T_\alpha T_\beta^*\right\}, \tag{3.19}$$

$$K_{yz}{}^{x}=\frac{2}{N}Im\left\{-UT_{\beta}{}^{*}+\frac{1}{2\sqrt{2}}\frac{1-\cos\theta}{\sin\theta}UT_{\alpha}{}^{*}+\sqrt{2}\cot\theta\ T_{\alpha}T_{\beta}{}^{*}\right\}, \tag{3.20}$$

$$K_{xz}{}^{y}=\frac{\sqrt{2}}{N}Im\left\{\left(U\cot\theta-2T_{\beta}\cot\theta-\frac{3}{\sqrt{2}}S\right)T_{\alpha}{}^{*}\right\}, \tag{3.21}$$

$$K_{xy}{}^{x}=-\frac{1}{\sqrt{2}N}Im\left\{UT_{\alpha}{}^{*}-US^{*}-4T_{\alpha}T_{\beta}{}^{*}+4ST_{\beta}{}^{*}-\frac{\sqrt{2}}{\sin\theta}ST_{\alpha}{}^{*}\right\}, \tag{3.22}$$

$$K_{yz}{}^{z}=\frac{1}{\sqrt{2}N}Im\left\{UT_{\alpha}{}^{*}+US^{*}-4T_{\alpha}T_{\beta}{}^{*}-4ST_{\beta}{}^{*}+\frac{\sqrt{2}}{\sin\theta}ST_{\alpha}{}^{*}\right\}, \tag{3.23}$$

$$K_{xx}{}^{y}=\frac{1}{N}Im\left\{\sqrt{2}\,UT_{\alpha}{}^{*}-\frac{\sqrt{2}}{3}US^{*}+2\sqrt{2}\ T_{\alpha}T_{\beta}{}^{*}\right.$$
$$\left.-\frac{2\sqrt{2}}{3}ST_{\beta}{}^{*}+\frac{1+9\cos\theta}{3\sin\theta}ST_{\alpha}{}^{*}\right\}, \tag{3.24}$$

$$K_{yy}{}^{y}=\frac{2\sqrt{2}}{3N}Im\left\{\left(U-2T_{\beta}+\frac{1}{\sqrt{2}\sin\theta}T_{\alpha}\right)S^{*}\right\}=A_{y}, \tag{3.25}$$

$$K_{zz}{}^{y}=\frac{1}{N}Im\left\{-\frac{\sqrt{2}}{3}US^{*}-\sqrt{2}\,UT_{\alpha}{}^{*}-\frac{2\sqrt{2}}{3}ST_{\beta}{}^{*}\right.$$
$$\left.+\frac{1-9\cos\theta}{3\sin\theta}ST_{\alpha}{}^{*}-2\sqrt{2}\ T_{\alpha}T_{\beta}{}^{*}\right\}, \tag{3.26}$$

$$K_{y}{}^{zz}=\frac{1}{N}Im\left\{-\frac{\sqrt{2}}{3}US^{*}+\sqrt{2}\,UT_{\alpha}{}^{*}-\frac{2\sqrt{2}}{3}ST_{\beta}{}^{*}\right.$$
$$\left.+\frac{1-9\cos\theta}{3\sin\theta}ST_{\alpha}{}^{*}+2\sqrt{2}\ T_{\alpha}T_{\beta}{}^{*}\right\}, \tag{3.27}$$

$$K_{xx}{}^{xz}=-\frac{1}{\sqrt{2}N}Re\left\{UT_{\alpha}{}^{*}-3US^{*}-2T_{\beta}T_{\alpha}{}^{*}+6T_{\beta}S^{*}\right.$$
$$\left.-\frac{3(1+\cos\theta)}{\sqrt{2}\sin\theta}T_{\alpha}S^{*}+\frac{9\cos\theta+1}{\sqrt{2}\sin\theta}|T_{\alpha}|^{2}\right\}, \tag{3.28}$$

$$K_{yy}{}^{xz}=\frac{1}{\sqrt{2}N}Re\left\{2UT_{\alpha}{}^{*}-4T_{\beta}T_{\alpha}{}^{*}-\frac{6\cos\theta}{\sqrt{2}\sin\theta}T_{\alpha}S^{*}\right.$$
$$\left.+\frac{\sqrt{2}}{\sin\theta}|T_{\alpha}|^{2}\right\}, \tag{3.29}$$

$$K_{xz}{}^{yy}=\frac{1}{\sqrt{2}N}Re\left\{2UT_\alpha{}^*-4T_\beta T_\alpha{}^*+\frac{6\cos\theta}{\sqrt{2}\sin\theta}T_\alpha S^*\right.$$
$$\left.+\frac{\sqrt{2}}{\sin\theta}|T_\alpha|^2\right\}, \tag{3.30}$$

$$K_{zz}{}^{xz}=\frac{1}{\sqrt{2}N}Re\left\{-UT_\alpha{}^*-3US^*+2T_\beta T_\alpha{}^*+6T_\beta S^*\right.$$
$$\left.-3\frac{1-\cos\theta}{\sqrt{2}\sin\theta}T_\alpha S^*+\frac{9\cos\theta-1}{\sqrt{2}\sin\theta}|T_\alpha|^2\right\}, \tag{3.31}$$

$$K_{yy}{}^{zz}=\frac{2}{N}Re\left\{-\frac{1}{6}|U|^2-\frac{2}{3}UT_\beta{}^*+\frac{1+3\cos\theta}{3\sqrt{2}\sin\theta}UT_\alpha{}^*\right.$$
$$-\frac{1}{4}|S|^2+\frac{3}{2}T_\alpha S^*+\frac{\cos\theta-1}{2\sin^2\theta}|T_\alpha|^2-2|T_\beta|^2$$
$$\left.+\sqrt{2}\frac{1-\cos\theta}{\sin\theta}T_\alpha T_\beta{}^*\right\}. \tag{3.32}$$

As was pointed out in the preceding section, the magnitude of U is the largest among those of all spin-space tensor amplitudes and thus the terms having U make large contributions to the physical quantities. However, as is seen in the above expressions, most of the quantities have two or more terms which include U, the effects of which are mixed up with each other. For making the effect of a particular spin-dependent interaction clear, it is useful to treat suitable linear combinations of the observables. The quantity $K_y{}^{zz}$, for example, includes US^* and $UT_\alpha{}^*$, the latter of which is a good measure of the effect of the T_R-type tensor interaction. To avoid the disturbance due to the US^* term, the following linear combination is useful.

$$K_y{}^{zz}+\frac{1}{2}A_y=\frac{1}{N}Im\{(\sqrt{2}\,U-3S\cot\theta-2\sqrt{2}\,T_\beta)T_\alpha{}^*\}. \tag{3.33}$$

This conclusion has been confirmed by numerical calculations in ref. 2.

§ 4. Summary

This note describes the method to decompose the deuteron scattering amplitude so that each component, the spin-space tensor amplitude, represents selectively the effect of a particular spin-dependent interaction. In particular, it is shown that the quantum-mechanical corrections to the classical concept of the angular momentum which has been used in the previous work are cancelled out with each other and thus the previous development is justified. The cross section, the analyzing powers and the polarization transfer coefficients are expressed in terms of the spin-space tensor amplitudes, the results of which represent the effect of each spin-dependent interaction explicitly. Using these, it is shown that the special linear combination of the observables can emphasize the effect of the particular spin-dependent interaction. We hope that these representations are useful in planning of experimental measurements as well as in the theoretical analyses of the experimental data.

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